TEMPERATURE DISTRIBUTION IN A POROUS MATRIX – LIQUID SYSTEM

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An approximate solution of the heat balance equations for the liquid and matrix in a porous matrix–liquid system is given. A comparison with the exact solution shows good agreement of the temperature profiles.

The solution of several practical problems requires a knowledge of the heat transfer coefficient for a flow of liquid through a packed bed. Such a need arises, in particular, in the planning of a number of electrical engineering installations.

The analysis of the complex problem of heat transfer is usually simplified by assuming that the thermophysical parameters are constant. The initial equations for analysis of heat transfer in a porous matrix and liquid are the heat balance equations. Radiant heat transfer is usually neglected in the investigations owing to the smallness of its contribution to the total transmitted heat flux in all the particular investigated conditions.

If there is no phase transition in the porous matrix–liquid system the heat balance equation for the porous matrix, which has porosity ε and in which heat energy is released uniformly throughout its volume, while the thermophysical parameters are constant and the liquid moves in one direction x, has the form

$$(1-\varepsilon)\rho_1 c_{p1}\frac{\partial t_1}{\partial \tau} = \lambda_1(1-\varepsilon)\frac{\partial^2 t_1}{\partial x^2} - \alpha_v(t_1 - t_2) + Q_v. \quad (1)$$

The heat balance equation for a liquid when the thermophysical parameters and flow velocity are constant has the form

$$\rho_2 c_{p2}\varepsilon\frac{\partial t_2}{\partial \tau} = \lambda_2\varepsilon\frac{\partial^2 t_2}{\partial x^2} + \alpha_v(t_1 - t_2) - \rho_2 c_{p2} v\frac{\partial t_2}{\partial x}. \quad (2)$$

The heat transfer coefficient is referred to unit volume of the porous matrix, which is common practice in several studies of the heat transfer associated with a flow through a porous matrix.

An analysis of this system was given by Green et al. [1] for boundary conditions of the first kind. These authors, unfortunately, could not obtain an analytical solution of system (1), (2). A numerical solution was obtained for several values of the system parameters but, as the authors themselves noted, the obtained solutions were rather complex for use in the evaluation of experimental results.

The above system of equations excluding the second term in the first equation was solved for the following boundary conditions:

$$t_1(0, \tau) = t_1', \; t_1(l, \tau) = t_2', \; t_1(x, 0) = t_1', \; t_2(0, \tau) = t_0', \; t_2(l, \tau) = t_2'. \quad (3)$$

Using the Laplace transformation we ultimately obtained very unwieldy expressions for t_1 and t_2, which we do not give here and which are rather inconvenient for practical use.

Since the heat transfer coefficient in our formulation of the problem is independent of x, and taking into account the prescribed boundary conditions, we introduce an approximate replacement of the temperature head by the product $(t_1' - t_0)(1 - xl^{-1})\exp[(-\lambda_2/\rho_2 c_{p2} l^2 v)x]$. We will assess the validity of this substitution later. The initial equations (1) and (2) then assume the form

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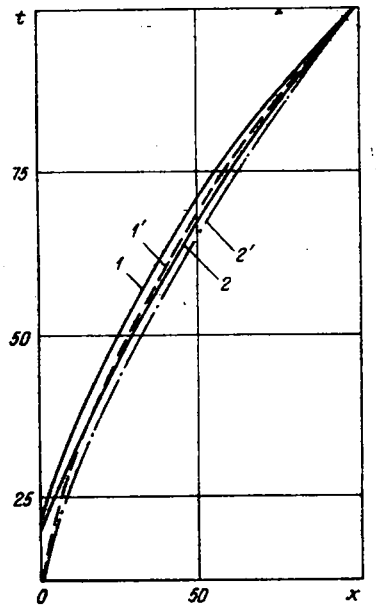

Fig. 1. Temperature distribution in porous matrix–liquid system obtained from exact and approximate solutions: 1) temperature of porous matrix from exact solution; 1') temperature of liquid from exact solution; 2) temperature of porous matrix from approximate solution; 2') temperature of liquid from approximate solution (t, °C, x, cm).

$$(1-\varepsilon)\rho_1 c_{p1}\frac{\partial t_1}{\partial \tau} = (1-\varepsilon)\lambda_1\frac{\partial^2 t_1}{\partial x^2} - \alpha_v k\left(1-\frac{x}{l}\right)\exp\left(-\frac{\lambda_2}{\rho_2 c_{p2} l^2 v}x\right) + Q_v, \tag{4}$$

$$\varepsilon\rho_2 c_{p2}\frac{\partial t_2}{\partial \tau} = \varepsilon\lambda_2\frac{\partial^2 t_2}{\partial x^2} + \alpha_v k\left(1-\frac{x}{l}\right)\exp\left(-\frac{\lambda_2}{\rho_2 c_{2p} l^2 v}x\right) - \rho_2 c_{p2} v\frac{\partial t_2}{\partial x}, \tag{5}$$

where $k = t_1' - t_0$ is determined from the boundary conditions.

The boundary conditions are the same, i.e., they are determined by system (3). The solution scheme for system (4), (5) is also the same. Applying Laplace transformations to (4), (5) we obtain

$$s\bar{t}_1 - t_1' = \frac{\lambda_1}{\rho_1 c_{p1}}\frac{d^2\bar{t}_1}{dx^2} - \frac{\alpha_v k}{(1-\varepsilon)\rho_1 c_{p1} s}\left(1-\frac{x}{l}\right)\exp\left(-\frac{\lambda_2}{\rho_2 c_{p2} l^2 v}x\right) + \frac{Q_v}{s\rho_1 c_{p1}(1-\varepsilon)}, \tag{6}$$

$$s\bar{t}_2 - t_0 = \frac{\lambda_2}{\rho_2 c_{p2}}\frac{d^2\bar{t}_2}{dx} - \frac{\alpha_v k}{\varepsilon\rho_2 c_{p2} s}\left(1-\frac{x}{l}\right)\exp\left(-\frac{\lambda_2}{\rho_2 c_{p2} l^2 v}x\right) - \frac{v}{\varepsilon}\frac{d\bar{t}_2}{dx}; \tag{7}$$

$$\bar{t}_1(0, s) = t_1'/s,\ \bar{t}_1(x, \infty) = t_1'/s,\ \bar{t}_1(l, s) = t_2'/s,\ \bar{t}_2(0, s) = t_0/s,$$
$$\bar{t}_2(l, s) = t_2'/s. \tag{8}$$

These transformations reduce the solution of the system of equations to the solution of a second-order equation. Since its solution presents no particular difficulties, and to avoid burdening our account with superfluous mathematical calculations, we give only the final expression for the temperatures

$$t_1(x, \tau) = \left[t_1' + \frac{Q_v}{(1-\varepsilon)\rho_1 c_{p1}}\tau\right]\left(1-\frac{x}{l}\right) + \frac{\alpha' k}{(a/vl^2)^2}$$
$$\times\exp(-ax/vl^2)\left\{1-\exp\left[\lambda'(a/vl^2)^2\tau\right]\right\}\left(1-\frac{x}{l}\right) - \frac{2a\alpha' k}{\lambda' l(a/vl^2)^2}\left\{1-\left[\lambda'(a/vl^2)^2\tau - 1\right]\exp\left[\lambda_1(a/vl^2)^2\tau\right]\right\}$$
$$\times\left\{\exp(-ax/vl^2) - \frac{x}{l}\exp(-a/vl)\right\} - \alpha'\left(k - \frac{2ak}{\lambda' vl^3}\right)$$
$$\times\int_0^{\tau_k}\left\{\exp\left[\lambda'(a/vl^2)^2\tau\right] - \tau\exp\left[\lambda'(a/vl^2)^2\tau\right]\right\}\left\{\operatorname{erfc}\left[\frac{x}{2\sqrt{\lambda'(\tau_k-\tau)}}\right]\right.$$
$$\left. - \frac{x}{l}\operatorname{erfc}\left[\frac{l}{2\sqrt{\lambda'(\tau_k-\tau)}}\right]\right\}d\tau - 2\int_0^{\tau_k}\left\{\operatorname{erfc}\left[\frac{x}{2\sqrt{\lambda'(\tau_k-\tau)}}\right] - \frac{x}{l}\operatorname{erfc}\left[\frac{l}{2\sqrt{\lambda'(\tau_k-\tau)}}\right]\right\}d\tau + \frac{x}{l}t_2', \tag{9}$$

and the liquid temperature is given by the expression

$$t_2(x, \tau) = t_1(x, \tau) - k\left(1-\frac{x}{l}\right)\exp(-ax/vl^2). \tag{10}$$

To assess the degree of approximation introduced by the assumption of an exponential variation of the difference in temperatures of the porous matrix and liquid with the coordinate we calculated the temperature profiles of the porous matrix and liquid from the exact system (1), (2) and the approximate system (9), (10) for the same conditions (Fig. 1).

The initial data were as follows: heat transfer medium – water, porous matrix – sintered aluminum, duration of experiment – 1000 sec, liquid temperature at entrance – 10°C, temperature of porous matrix – 20°C, liquid temperature at exit – 100°C, bulk heat release – 0.08 J/cm^3 · sec. The maximum difference in the temperatures calculated from the exact and approximate solutions was 4%.

From this we can conclude that the approximate solution (9), (10), which is simpler than the exact solution, can be recommended for evaluation of temperatures in a porous matrix–liquid system.

NOTATION

ε is the porosity;
λ is the thermal conductivity;
ρ is the density;
c_p is the heat capacity;
t is the temperature;
τ is the time;
x is the coordinate;
α_v is the heat transfer coefficient per unit volume of porous matrix;
Q_v is the heat flux per unit volume of porous matrix;
v is the liquid filtration velocity;
l is the thickness of porous matrix in direction of coordinate;
$\alpha' = \alpha_v/\rho_1 c_{p1}(1-\varepsilon)$;
a is the thermal diffusivity;
$\lambda' = \lambda_1/\rho_1 c_{p1}$.

Subscripts

1 denotes porous matrix;
2 denotes liquid.

LITERATURE CITED

1. D. W. Green, R. H. Perry, and R. E. Babcock, A. I. Ch. E. J., 10, No. 5, 645 (1964).